Synchronous flashing of fireflies

To elucidate the mechanism for synchronous flashing of fireflies

1. To obtain information about the emission patterns, by introducing the coupling strength dependent on the distance into Kuramoto model
2. To consider the effect when the random walk is added to the model

Model of synchronization

\[ \frac{d\theta_i}{dt} = \omega_i + \frac{K}{N} \sum_{j=1}^{N} \sin(\theta_j - \theta_i) \]

\( \omega_i \) and \( \theta_i \) are the natural frequency and the phase of the \( i \)-th oscillator, respectively.

New model

\[ \frac{d\theta_i}{dt} = \omega_i + \frac{K}{N} \sum_{j=1}^{N} \sin(\theta_j - \theta_i) \]

\( M = \sum_{j=1}^{N} \sin(\theta_j - \theta_i) \)

\( D: \) Interacting distance

\( \theta_i \) is the natural frequency and \( \phi_i \) is the phase of the \( i \)-th oscillator.

Process up to the synchronous state

Simulation Step 1

Population of fireflies is 165 (11 × 15)
Each individual which is given one specific frequency (1 ~ 1.25Hz)
Initial state: Random phase (0 ~ 2\( \pi \))
Grid-like position
Motion of firefly: Fixed and random walk

Simulation Step 2

Population of fireflies 165 (11 × 15)
All individuals which are given one specific frequency (1Hz)
Initial state: Random phase (0 ~ 2\( \pi \))
Grid-like position
Motion of firefly: Fixed and random walk
Definition of the degree of synchronization: 

\[ \sigma = \exp(0) = \exp \left( \sum_{i=1}^{N} \phi_i \right) \]

Discussion

When \( D = 0.5 \), the effect of the random walk is clearly seen.

As the interaction distance is smaller, more significant difference between the fixed condition and the random walk was shown.

While it takes a longer time up to the synchronous state if the random walk is used, high reliability could be observed.

Summary

1. Complex emission patterns like a spiral or a propagating wave were obtained by introducing the coupling strength dependent on the distance into the model.
2. Significant difference was observed in the synchronization process by the addition of the random walk

Future tasks

1. Study of a movement different from a random walk
2. Investigation for the effects of a spiral or a propagating wave in the synchronization process