## Effect of random walk and dependence on coupling length for synchronous flashing of fireflies in computer simulation

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Graduate School of Science and Engineering,
    Yamaguchi University
        Shun Takemoto
        Yuko Nagamine
        Hidetoshi Miike
        Atsushi Osa

\section*{Synchronous flashing of fireflies}
- In Southeast Asia, fireflies flock in one tree and flash light in the same period all together.
- Complex emission patterns like a spiral and a propagating wave have been confirmed \({ }^{(1)}\)


Figure 1 Firefly tree
http://www.mech.usp.ac.jp/~hnw/theme/bunnya_2007/hikikomi_hotal.html
(1) [Symphony of Light tropical forest ] 13: March 2004 NHK [Earth! Mysterious nature] Broadcasted by TV

\section*{Synchronization}
- Repeat of the same process is called rhythm phenomenon, and entrainment that occurs between the rhythm and the rhythm is defined as SYNCHRONIZATION.
- Coherent phenomenon among individual oscillators occurs when more than two interacting oscillators couple.

\section*{Problems in Kuramoto model}
- As conventional Kuramoto model ignores the distance effect, some problems occur.

- Using this model, emission patterns, such as a spiral and a propagating wave, can not be explained.

\section*{Purpose}
- To elucidate the mechanism for synchronous flashing of fireflies
1. To obtain information about the emission patterns, by introducing the coupling strength dependent on the distance into Kuramoto model
2. To consider the effect when the random walk is added to the model

\section*{Model of synchronization}

Kuramoto model \({ }^{(2)}\)
\[
\frac{\mathrm{d} \varphi_{\mathrm{i}}}{\mathrm{dt}}=\omega_{\mathrm{i}}+\frac{\mathrm{K}}{\mathrm{~N}} \sum_{\mathrm{j}=1}^{\mathrm{N}} \sin \left(\varphi_{\mathrm{j}}-\varphi_{\mathrm{i}}\right)
\]
\(\omega i\) and \(\varnothing i\) are the natural frequency and the phase of the i-th oscillator, respectively.
New model
\[
\begin{gathered}
\frac{\mathrm{d} \varphi_{\mathrm{i}}}{\mathrm{dt}}=\omega_{i}+\frac{\mathrm{K}}{\mathrm{M}} \sum_{\mathrm{j}=1}^{\mathrm{N}} \alpha_{\mathrm{j}} \cdot \sin \left(\varphi_{\mathrm{j}}-\varphi_{\mathrm{i}}\right) \\
\mathrm{M}=\sum_{\mathrm{j}=1}^{\mathrm{N}} \alpha_{\mathrm{j}} \quad \begin{array}{l}
\alpha_{\mathrm{j}}=1 \text { (if the distance between } \mathrm{i} \text { and } \mathrm{j} \text { is less than or equal to } \mathrm{D} \text { ) } \\
\alpha_{\mathrm{j}}=0 \text { (if the distance between } \mathrm{i} \text { and } \mathrm{j} \text { is longer than } \mathrm{D} \text { ) } \\
\mathrm{D}: \text { Interacting distance }
\end{array}
\end{gathered}
\]
(2) Kuramoto Yoshiki (2007) About the so-called "Kuramoto model" 17(2), 175-177

\section*{Simulation Step 1}
- Population of fireflies is \(165(11 \times 15)\)
- Each individual which is given each specific frequency ( \(1 \sim 1.25 \mathrm{~Hz}\) )
- Initial state: Random phase ( \(0 \sim 2 \pi\) )

Grid-like position
- Motion of firefly: Fixed and random walk



\begin{tabular}{|c|c|}
\hline \multicolumn{2}{|l|}{\begin{tabular}{l}
Random walk effect in the process of synchronization \\
A statistical result (standard deviation <0.4) for time up to synchronous state
\end{tabular}} \\
\hline Distance of interaction & Degree of Difference \\
\hline Small( \(\mathrm{D}=2\) ) & Significant difference
\[
(p=0.02)
\] \\
\hline Middle(D=5) & Some difference
\[
(p=0.66)
\] \\
\hline \(\operatorname{Big}(\mathrm{D}=8)\) & no difference ( \(\mathrm{p}=0.97\) ) \\
\hline
\end{tabular}

\section*{Simulation Step 2}
- Population of fireflies 165 (11 \(\times 15\) )
- All individuals which are given one specific frequency ( 1 Hz )
- Initial state: Random phase ( \(0 \sim 2 \pi\) )

Grid-like position
- Motion of firefly: Fixed and Random walk
- Definition of the degree of synchronization
\[
\sigma \exp (\mathrm{i} \theta)=\mathrm{N}^{-1} \sum_{\mathrm{j}=1}^{\mathrm{N}} \exp \left(\mathrm{i} \varphi_{\mathrm{j}}\right)
\]


\section*{Summary}
1. Complex emission patterns like a spiral or a propagating wave were obtained by introducing the coupling strength dependent on the distance into the model.
2. Significant difference was observed in the synchronization process by the addition of the random walk

\section*{Future tasks}
1. Study of a movement different from a random walk
2. Investigation for the effects of a spiral or a propagating wave in the synchronization process

\section*{Discussion}
- When \(\mathrm{D}=0.5\), the effect of the random walk is clearly seen.
- As the interaction distance is smaller, more significant difference between the fixed condition and the random walk was shown.
- While it takes a longer time up to the synchronous state if the random walk is used, high reliability could be observed```

